Micro C - Spring 2014 - Exam Solutions

1. Consider the following game G, where Player 1 chooses the row and Player 2 simultaneously chooses the column.

		Player 2		
		D	E	F
	A	$3,5$	$3,4$	$4,2$
Player 1	B	$2,3$	$x,4$	$3,2$
	C	$1,2$	$1,1$	$1,5$

   (a) Is G a static game or a dynamic game?

   SOLUTION: By definition, G is a static game.

   (b) Suppose $x = 4$. Find all Nash equilibria (pure and mixed) in G.

   SOLUTION: The pure strategy Nash Equilibria are (A, D) and (B, E). They can be found by underlining the highest payoff for Player 1 in each row and for Player 2 in each column. Notice that B strictly dominates C, and after C is removed from the game, E strictly dominates F. Hence, in a mixed strategy Nash Equilibrium, Player 1 must be indifferent between A and B, and Player 2 must be indifferent between D and E. This is the case if Player 1 plays A and B each with probability $1/2$ (so that Player 2 earns 4 from both D and E), and Player 2 plays D and E each with probability $1/2$ (so that Player 1 earns 3 from both A and B).

   (c) Suppose $x = 2$. Find a Nash equilibrium in G and prove that it is unique. Make sure to explain your reasoning (2-3 sentences).

   SOLUTION: The strategy profile (A, D) is a Nash Equilibrium. To prove that this Nash Equilibrium is unique, it is enough to show that no other strategy profile survives Iterated Elimination of Strictly Dominated Strategies: B dominates C, then E dominates F, then A dominates B, then D dominates E.

2. Consider a market for pies with 2 bakers. Bakers i and j follow the same recipe, and are highly skilled at pie-making, so they both produce identical pies at zero marginal cost. They simultaneously and independently set prices p_i and p_j. Demand for baker i is 0 whenever $p_i > 1$. For any price $p_i \leq 1$, demand for baker i is $q_i = D$ when $p_i < p_j$ and $q_i = D/2$ when $p_i = p_j$, where $D > 0$ is a strictly positive constant. In words, the baker with the lowest price serves the entire market, but demand drops to zero when the price exceeds 1. Profits for baker i are $\pi_i = p_i q_i$.

   In this question, you only have to consider symmetric equilibria, where both bakers set the same price, and earn the same profits.

   (a) Suppose the market for pies operates for only one period. Solve for a Nash Equilibrium where both firms earn zero profits, $\pi_i = \pi_j = 0$. Is this equilibrium unique? Why can't firms collude on the monopoly price, $p = 1$? Explain your answers briefly (2-3 sentences).

SOLUTION: Firms set $p_i = p_j = 0$ and earn zero profits. Neither firm has an incentive to deviate, because setting a strictly positive price would result in zero demand. This equilibrium is unique. At any other (positive) prices, one firm can strictly profit by marginally undercutting the other firm and stealing the entire market. This explains why firms cannot collude on the monopoly price.

(b) Suppose the market operates for two periods. Solve for the Subgame Perfect Nash Equilibrium that gives the highest profits to the bakers. Comment briefly (1-2 sentences).

SOLUTION: There is always a Subgame Perfect Nash Equilibrium where players repeat the Nash Equilibrium of the stage game in each period. Here, firms set $p_i = p_j = 0$ in period 1, and $p_i = p_j = 0$ in period 2, regardless of the prices (p_i, p_j) from period 1. This Subgame Perfect Nash Equilibrium is unique, since $\pi_i = \pi_j = 0$ is the unique Nash Equilibrium of the stage game. Firms realize that it is not credible to promise positive prices in period 2, leaving them unable to collude on positive prices in period 1.

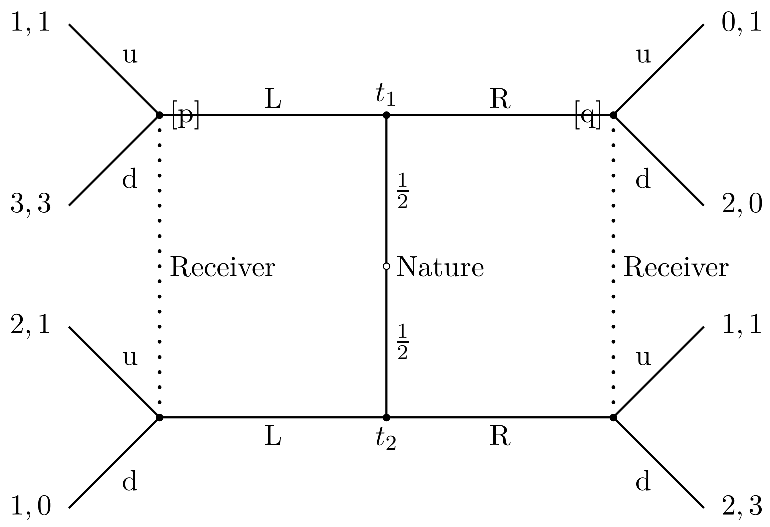
(c) Suppose the market operates for infinitely many periods, and bakers have discount factor $\delta < 1$. Solve for the Subgame Perfect Nash Equilibrium that gives the highest (per period) profits to the bakers. How do these profits compare to your answers in parts (a) and (b)? Explain briefly (2-3 sentences).

SOLUTION: Consider trigger strategies, where firms set $p_i = p_j \equiv p \leq 1$ in period 1. In any period $t \geq 2$, they set $p_i = p_j = p$ if this was the outcome in all earlier periods, and otherwise they set $p_i = p_j = 0$. Equilibrium payoffs are $p(D/2)/(1-\delta)$. If one firm slightly undercuts the other, it earns pD, and then 0 as it is punished in all later periods. This deviation is profitable if $pD > p(D/2)/(1-\delta)$, where simplifying gives $\delta < 1/2$. Thus, if $\delta < 1/2$, then the unique Subgame Perfect Nash Equilibrium has firms always set $p_i = p_j = 0$. If $\delta \geq 1/2$, then the most profitable Subgame Perfect Nash Equilibrium has firms play trigger strategies described above, with $p_i = p_j = 1$. For firms to earn positive profits, they must be sufficiently patient, so that the current gain from undercutting a rival is outweighed by the future loss from the punishment.

(d) In most real-world markets, increased competition tends to drive down prices. Is this the case in the market for pies? Specifically, comment on how the results in parts (a), (b) and (c) might change if there were three bakers in the market (3-4 sentences).

SOLUTION: The market for pies describes an extreme situation where products are perfect substitutes, so that Bertrand competition with two firms already drives price down to marginal cost. The answers in part (a) and (b) would not change if there were 3 bakers in the market because prices cannot drop below zero. In part (c), prices might drop, because collusion would become more difficult. With three bakers, each one would earn lower equilibrium profits (from serving 1/3 < 1/2 of the market), making it more difficult to sustain an equilibrium with positive prices.

3. Now consider the following game G':



(a) Is G' a game of complete or incomplete information?

SOLUTION: By definition, G is a game of incomplete information.

(b) Find two pooling equilibria in G': one where both sender types play L, and another where both sender types play R.

SOLUTION: Pooling equilibria: $(LL, du, p = 1/2, q \geq 2/3)$ *and* $(RR, ud, p \leq 1/3, q = 1/2)$.

(c) Check whether or not each pooling equilibrium in (b) satisfies Signaling Requirement 5.

SOLUTION: All the pooling equilibria satisfy Signaling Requirement 5, since no sender type has a strictly dominated strategy.

(d) Which pooling equilibrium in part (b) seems most reasonable? Explain your answer briefly using concepts from the course (2-3 sentences).

SOLUTION: Pooling on R might seem more reasonable than pooling on L, since the equilibria $(LL, du, p = 1/2, q \geq 2/3)$ do not satisfy Signaling Requirement 6. This requirement imposes $q = 0$, since the message R for type 1 is equilibrium dominated by L.

4. Anna and Bo must decide how to share three liters of wine. Their preferences for wine are given by $u_A(x_A) = x_A^{1/3}$ and $u_B(x_B) = x_B^{1/3}$, where x_A is the amount of wine that Anna receives, and x_B is the amount of wine that Bo receives. If they fail to reach an agreement, they both get nothing.

(a) Think of this situation as a coalitional game. What is the set of possible coalitions? Which allocations are in the core?

SOLUTION: There are three possible coalitions: $\{Anna\}$, $\{Bo\}$, $\{Anna, Bo\}$. *The* *core consists of allocations with $x_A \geq 0$, $x_B \geq 0$, and $x_A + x_B = 3$, since the coalitions $\{Anna\}$ and $\{Bo\}$ have zero worth, and since the grand coalition $\{Anna, Bo\}$ can improve on any allocation with $x_A + x_B < 3$.*

(b) Think of this situation as a bargaining problem. What is the Nash bargaining solution?

SOLUTION: The bargaining problem is (U, d), where $U = \{(v_A, v_B) : v_A, v_B \geq 0, v_A^3 + v_B^3 \leq 3\}$, and $d = (0, 0)$. Applying PAR and SYM, the Nash bargaing solution is $v_A = v_B = (3/2)^{1/3}$.

(c) In this particular situation, are all four axioms (PAR, SYM, INV, and IIA) necessary to arrive at the Nash bargaining solution? Briefly explain your answer (2-4 sentences).

SOLUTION: PAR eliminates any (v_A, v_B) that is not Pareto efficient, $v_A^3 + v_B^3 < 3$. SYM eliminates any (v_A, v_B) that is not symmetric, $v_A \neq v_B$. This is sufficient to give the Nash bargaining solution. Axioms INV and IIA are not necessary in this situation.